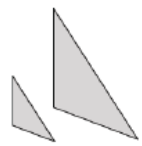
Similarity

In this 14-lesson module, students learn about dilation and similarity and apply that knowledge to a proof of the Pythagorean Theorem based on the Angle-Angle criterion for similar triangles. Students learn the definition of a dilation, its properties, and how to compose them. One overarching goal of this module is to replace the common idea of "same shape, different sizes" with a definition of similarity that can be applied to shapes that are not polygons, such as ellipses and circles.

Two geometric figures are said to be similar if they have the same shape but not necessarily the same size. Using that **informal definition**, are the following pairs of figures similar to one another? Explain.

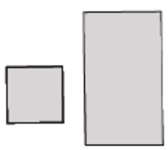
Solution:



Yes, these figures appear to be similar. They are the same shape but one is larger than the other is, or one is smaller than the other is.

Two geometric figures are said to be similar if they have the same shape but not necessarily the same size. Using that **informal definition**, are the following pairs of figures similar to one another? Explain.

Solution:



No, these figures do not appear to be similar. One looks like a square and the other like a rectangle.

What Came Before this Module:

Students learned about translations, reflections, and rotations in the plane and how to use them to precisely define the concept of *congruence*. Students were also introduced to the Pythagorean Theorem.

What Comes After this Module:

Students extend what they already know about unit rates and proportional relationships to linear equations and their graphs. They will understand the connections between proportional relationships, lines, and linear equations. In addition, students will apply the skills they acquired in Grades 6 and 7, with respect to symbolic notation and properties of equality to transcribe and solve equations in one variable and then in two variables.

Key Words

Dilation: A transformation of the plane with center O and scale factor $r (r > 0)$. If $D(O) = O$ and if $P \neq O$, then the point $D(P)$, to be denoted by Q, is the point on the ray OP so that $|OQ| = r|OP|$. If the scale factor $r \neq 1$, then a dilation in the coordinate plane is a transformation that shrinks or magnifies a figure by multiplying each coordinate of the figure by the scale factor.

Congruence: A finite composition of basic rigid motions—reflections, rotations, translations—of the plane. Two figures in a plane are *congruent* if there is a congruence that maps one figure onto the other figure.

Similar: Two figures in the plane are *similar* if a similarity transformation exists, taking one figure to the other.

Similarity Transformation: A *similarity transformation*, or *similarity*, is a composition of a finite number of basic rigid motions or dilations. The scale factor of a similarity transformation is the product of the scale factors of the dilations in the composition; if there are no dilations in the composition, the scale factor is defined to be 1.

Similarity: A *similarity* is an example of a transformation.

How can you help at home?

- ✓ Every day, ask your child what they learned in school and ask them to show you an example.
- ✓ Ask your child why "same shape, different sizes" is not appropriate anymore when describing similarity.
- ✓ Ask your child to create an angle using a ruler. Have your child demonstrate how to measure that angle using a protractor.

Key Common Core Standards:

Understand congruence and similarity using physical models, transparencies, or geometry software.

- Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
- Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
- Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.

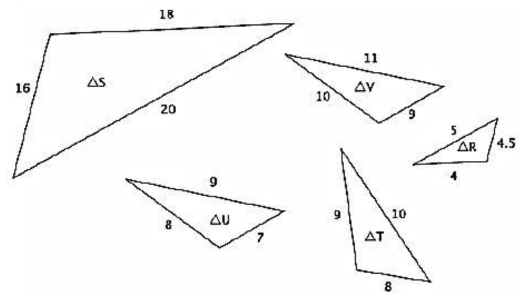
Understand and apply the Pythagorean Theorem.

- Explain a proof of the Pythagorean Theorem and its converse.
- Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

Symmetric and Transitive Relationships in Similarity

- Similarity is a symmetric relation. That means that if one figure is similar to another, $S \sim S'$, then we can be sure that $S' \sim S$. The sequence that maps one onto the other will be different, but we know that it is true.
- Similarity is a transitive relation. That means that if we are given two similar figures, $S \sim T$, and another statement about $T \sim U$, then we also know that $S \sim U$. Again, the sequence and scale factor will be different to prove the similarity, but we know it is true.

Use the diagram below to answer Questions 1 and 2.



Solution:

$\triangle \boldsymbol{S} \sim \triangle \boldsymbol{R}$ and $\triangle \boldsymbol{R} \sim \triangle \boldsymbol{S}$.

$\triangle \boldsymbol{S} \sim \triangle \boldsymbol{T}$ and $\triangle \boldsymbol{T} \sim \triangle \boldsymbol{S}$.

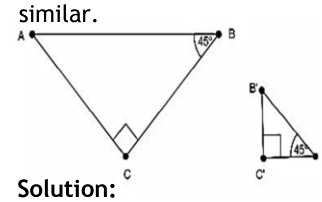
$\triangle \boldsymbol{T} \sim \triangle \boldsymbol{R}$ and $\triangle \boldsymbol{R} \sim \triangle \boldsymbol{T}$.

2. Which three triangles, if any, have similarity that is transitive?

One possible solution: Since $\triangle S \sim \triangle R$ and $\triangle R \sim \triangle T$, then $\triangle S \sim \triangle T$.

Note that $\triangle U$ and $\triangle V$ are not similar to each other or any other triangles. Therefore, they should not be in any solution.

Are the triangles shown below similar? Present an informal argument as to why they are or are not similar.



Solution:

Yes, $\triangle \boldsymbol{ABC} \sim \triangle \boldsymbol{A'B'C'}$. They are similar because they have two pairs of corresponding angles that are equal. You have to use the triangle sum theorem to find out that $|\angle \boldsymbol{B'}| = \boldsymbol{45}°$or $|\angle \boldsymbol{A}| = \boldsymbol{45}°$. Then you can see that $|\angle \boldsymbol{A}| = |\angle \boldsymbol{A'}| = \boldsymbol{45}°$, $|\angle \boldsymbol{B}| = |\angle \boldsymbol{B'}| = \boldsymbol{45}°$, and $|\angle \boldsymbol{C}| = |\angle \boldsymbol{C'}| = \boldsymbol{90}°$.

Pythagorean Theorem

Check out this video that demonstrates this proof using similar triangles!

http://www.youtube.com/watch?v=QCyvxYLFSfU

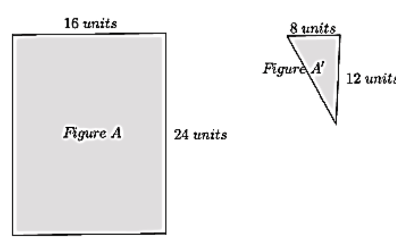
Fun Geometry Fact!

The word 'geometry' comes from the Greek words 'geo,' meaning earth, and 'metria,' meaning measure.

Knowing that a dilation followed by a *congruence* defines similarity helps determine if two figures are, in fact, similar. For example, would a dilation map Figure A onto Figure A′?

(i.e., Is Figure A ~ Figure A′?)

Images:



Solution::

No, even though we could say that the corresponding sides are in proportion, there exists no single rigid motion or sequence of rigid motions that would map a four-sided figure to a three-sided figure. Therefore, the figures do not fulfill the congruence part of the definition for similarity, and Figure A is not similar to Figure A'.